1. Protocol to study the parameters of the VCM

It is important to study how to set the parameters for our estimator in the context of varying-diameter tubes. Our first experiment aims at studying interesting r values for tubes with different radius ρ. The protocol is the following. A curve has been generated according to the following parametric equation: $x(t) = t$, $y(t) = t$, $z(t) = log(t)$.

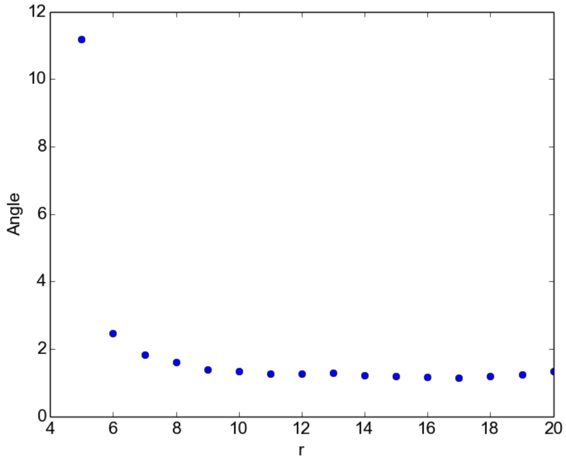

Figure 1: Angle difference between theoretical and computed tangent as a function of the integration radius r for a tubular volume or radius $\rho = 5$. The optimal r correspond to the first r value for which the curve plateaus (equal to 7 in this case).

Various volumes have been generated by fitting balls of fixed radius ρ along the curve. Then, orthogonal planes are estimated at the points of the parametric curve. At each point of the curve, the theoretical tangent can be computed. The difference in angle between the computed tangent (i.e. the orthogonal plane normal) and the theoretical tangent is measured for varying integration radius r. The optimal r is the first value for which the curve plateaus (cf. Fig 1). Table 1 shows the average of the optimal r values obtained at each centerline point.

Table 1: Best values for r and R for different volumes defined by ρ: curve ($\rho = 0$), and varying-diameter tubes (from $\rho = 1$ to $\rho = 10$). The results show the radius for the domain of integration r is dependent on the tube radius ρ.

Volume ($\rho =$)	0	1	2	3	4	5	6	7	8	9	10
Optimal r	5	5	5	5	6	7	8	9	10	11	12
Optimal R	7	7	7	7	7	7	7	7	7	7	7

On the **curve** ($\rho = 0$), the best value for r is 5, for which the maximal difference in angle with the theoretical tangent is 2 degrees. In the case of a curve, r should be chosen to take into account enough neighbors so that the orthogonal plane normal estimation is not sensitive to irregularities.

For the **volumes** with $\rho > 2$, the optimal value for r is directly dependent on the radius of the tube. Intuitively, all surface points corresponding to the cross-sectional plane at a given point should be taken into account in the domain of integration. Choosing $r = \rho$ is not sufficient due to digitization effects. This is confirmed by the experimentation which gives an optimal value for r when it is equal to $\rho + 2$. This does not hold for low radius tubes with $\rho \leq 2$. For these tubes, the integration radius r is larger in order to take into account a greater neighborhood, due to the low resolution.

This is true for orthogonal plane estimation at points centered in the volume, that is to say curve-skeleton points. Now, we study how to tune r for non-centered points in the volume. In the following, we choose the same integration radius r for all volume points which are at the same distance to the border of the tube. The protocol is the following: (a) create groups of points with the same distance transform (DT) value (b) for each group G, use different values for r (c) estimate the orthogonal plane normal with the VCM at each point $g \in G$ and (d) identify the best value for r as the radius which minimizes the difference in angle between the estimated normal and the expected one. The expected orthogonal plane is the plane orthogonal to the centerline (curve) and intersecting g. In the following, we used the tube with $\rho = 7$ for our experiments.

Table 2: Best values for r on the tube with $\rho = 7$ for non-centered points.

DT value	1	2	3	4	5	6
Optimal r	16	15	14	13	11	10

Table 2 shows the average of the optimal r values obtained at points with the same DT values. The closer a point is to the surface (low DT values), the higher the values for r. This can be explained because all surface points which Voronoi cells are aligned with the orthogonal plane are necessary for the computation. In other words, when a point is close to the surface, the surface points on the other side of the tube should be taken into account in the domain of integration of the VCM. Thus, for a given non-centered point in the volume, it is not possible to set r to a predetermined value like for centered points.

The observation that all surface points must be taken into account is a problem for points located in areas of high-curvature. Indeed, since the domain of integration is a ball, it would take too much points into to account, to properly consider the local curvature. Thus, we assume this observation holds for tubular volumes with low curvature.

The same study has been conducted for values of R. The results in Table 1 show sufficient size is obtained for the same values of R regardless of the volume. The ideal value for R is independent of the tube radius ρ. Informally, R should be large enough so that it accounts for digitization effects but small enough to integrate local information.